Golf Ball Flight Dynamics

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Introduction

Although poorly documented, golf is believed to have originated in Scotland in 1456[1]. It was first played as a very casual game for which no standard rules existed. A wooden ball was used in conjunction with wooden clubs prior to 1618[2], when the "featherie" (a ball made of stitched leather and tightly packed with feathers) was introduced. The featherie was favored for its more forgiving feel on the hands of players when it was struck and was used until 1848 when the invention of the "Gutta" surpassed the "featherie" in both durability and cost. The "Gutta" was made of gutta-percha packing material which was not brittle and became soft and moldable at 100°C.

The Gutta's pliability made it necessary to roll the ball on a "smoothing board" in order to maintain its shape and keep it free of imperfections which were created during normal play of the game. The smooth Gutta was used for only a few years before players began to realize that balls that had not been well maintained and had many nicks and scratches had a much more favorable flight. Thus began the practice of hammering the Gutta with a sharp-edged hammer in a regular pattern to increase the consistency of the ball's play.

In 1898 the first "Balata" ball was created by wrapping rubber thread around a solid rubber core which was then covered by a solid layer of rubber that later became known as the "ball cover". The Balata was the first sign of a modern age of golf technology for it allowed molds to be used to create consistent cover patterns. In 1908 makers discovered the superiority of a regular "dimple" pattern over the haphazard grid pattern favored by players at the time.

Dimples are small indentations on the exterior of the golf ball. They are typically round in shape and vary in diameter from 2-5mm in diameter and are about .2mm deep. Modern golf balls pack anywhere from 300-450 dimples of varying size arranged in a regular pattern on the outside of every ball[3]. Dimples have been one of the most influential developments in golf ball design because they alter the dynamics of the balls flight in such a way that gives golfers a significant amount of control over the height and shape of their shots.

Today, golf balls are highly regulated by the United States Golf Association (USGA) for compliance with rules governing the design and capabilities of a golf ball. Modern materials, such as Surlyn and Urethane as well is different core designs, have given ball designers the ability to create golf balls with almost any property desired (higher spin, softer feel at impact, lower trajectory, etc...). It is well known that due to the complexities involved with the aerodynamics of a golf ball, ball designers have taken a "design, test, and modify" approach to ball design as an alternative to computer modeling and CAD techniques.

In this paper I investigate a recent mathematical derivation for the aerodynamics of a smooth sphere and attempt to extend it to golf balls by incorporating the effects of dimples on air flow and comparing the results of simulations to observations taken from modern professional golfers. Although this technique will hopefully one day lead to the derivation of equations governing the flight of balls with arbitrary dimple patterns in pursuit of optimizing dimple design for maximum distance, the scope of this paper is limited to comparing two key effects that dimples have on ball flight.

Terminology

Golfers use a significant vocabulary that may not be recognized by those unfamiliar with the game. This section gives an introduction to basic terminology that will be used throughout this document.

Loft - The angle between the club face and the shaft. The more loft a club has, the higher it will launch the ball at impact.

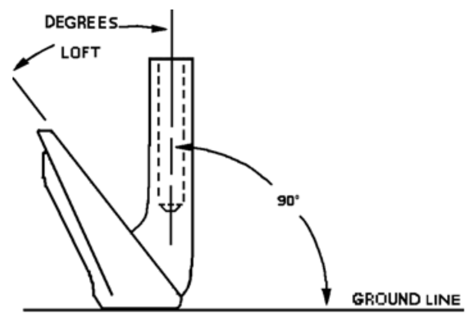

Figure 1: Loft[4]

Impact - The instant in time when a player strikes the ball with the club.

Ball flight - The path a ball takes after impact, while it is in the air. There are also two broad types of shot *shapes*: a *hook* and a *slice*, and two directional descriptors: a *pull* and a *push*. A hook is a ball flight in which the ball curves from right to left due to a small amount of side-spin being imparted to it at impact. Similarly, a slice is a ball flight in which the ball curves from left to right, due to sidespin imparted to the ball at impact in the opposite direction from the hook result. A pull is when the ball starts its path to the left of its intended destination. Likewise, push is when the ball starts its path to the right of its intended destination. Please note that each of these terms is defined for a player using "right-handed" clubs, and changes its meaning if the player uses left-handed clubs.

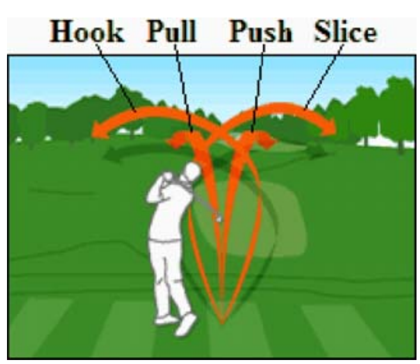

Figure 2: Ball Flights[5]

Carry - The distance a ball travels in the air, after being struck. Note that this excludes the distance that the ball bounces and rolls after it first strikes the ground (this is called *roll*).

Idealizing a Golf Ball: Playing with a Smooth Sphere

Introduction

A golf ball is a sphere about 4.2cm in diameter, with hundreds of small niches (dimples) carved in its outer face. While the dimples play a key role in the flight dynamics of the ball, I refrain from covering their effects until the next section. In this section we use a complex mathematical model derived and published by Karl I. Borg[6] in 2003 to model the flight of a golf ball under conditions typical for a professional golfer when striking the ball with a driver (see table 1). While this model is intended for use at low Reynolds Numbers, it has been experimentally shown to be reasonable as well for higher Reynolds Numbers. The Reynolds Number is a quantity that indicates the relative importance of inertial forces compared to viscous forces in determining its path through a gas or fluid. High Reynolds Numbers indicate that viscous forces are less important.

$$Re = \frac{d \cdot V}{\nu} \tag{1}$$

Equation (1) is used to compute the Reynolds Number (Re), where d is the diameter of the object, V is the objects velocity, and ν is kinematic viscosity of the fluid or gas, given by (2),

$$\nu = \frac{\mu}{\rho} \tag{2}$$

where μ is the Absolute Viscosity of the fluid or gas (fixed value for a given compound) and ρ is its density. For a golf ball flying through 68°F air at sea level, the associated Reynolds Number falls in the range of 2.15x10^5 to 9.0x10^4, which means that viscosity is much less important than the ball's inertia.

The Model

Borg's model incorporates drag, heat exchange between the gas and the sphere (ball), and the Magnus Effect. The Magnus Effect (occasionally referred to as the Robins Effect for spheres) is a lift force that results from the rotation of a cylinder or sphere as it moves through a fluid or gas and was first described by German physicist Heinrich Magnus in 1853[7]. The lift force occurs as a result of spin creating a region of lower pressure above a ball with backspin (due to Bernoulli's Principle). The net force on the sphere as derived by Borg et al is given by (3)

$$\vec{\mathbf{F}} = -\alpha_\tau \frac{\pi}{12} pR^2 \sqrt{\frac{\pi m}{2k_B T}} \chi \Re(z) \vec{\mathbf{v}} - \alpha_\tau \xi \frac{2}{3} \pi R^3 mn \vec{\omega} \times \vec{\mathbf{v}} - \alpha_\tau \frac{1}{3} \pi R^3 mn \chi \frac{\Im(z)}{|\vec{\omega}|} \vec{\omega} \times (\vec{\omega} \times \vec{\mathbf{v}}) \tag{3}$$

where α_τ is the accommodation coefficient of tangential momentum (fraction of reflected gas particles that are reflected diffusely), R is the radius of the sphere, m is mass of a single molecule of the gas, k_T is Boltzmann's Constant, T is the temperature of the gas, $\vec{\omega}$ is the angular velocity of sphere's rotation, $\vec{\mathbf{v}}$ is the velocity of the sphere, and p, χ, z, and ξ are given by equations (4) through (8)

$$p = nk_B T \tag{4}$$

$$\chi = \alpha_\tau \frac{nk_B T}{\kappa}\sqrt{\frac{k_B T}{2\pi m}} \tag{5}$$

$$B = \frac{1-i}{\sqrt{2}}\sqrt{\frac{|\vec{\omega}|R^2}{k}} \tag{6}$$

$$z = \frac{j_1(B)}{\chi j_1(B) + B j_1'(B)} \tag{7}$$

$$\xi = 1 - \frac{\chi}{4}\left[\Re(z) + \sqrt{\frac{2\pi k_B T}{m}}\frac{\Im(z)}{4|\vec{\omega}|R}\right] \tag{8}$$

where n is the number density of the gas, κ is the heat conductivity of the sphere, j_1 are the spherical Bessel functions of the first kind (with j_1' its derivative), $i = \sqrt{-1}$, and k is given by

$$k = \frac{\kappa}{\rho C_p} \tag{9}$$

where C_p is the heat capacity of the sphere and ρ is the density of the sphere. Values used in the simulation for each of these constants can be found in appendix A. Many of the parameters required by this model are properties of materials whose values are readily available, while others are determined based on launch conditions. For purposes of comparison and the availability of data, launch conditions for tour professionals were used where required - launch conditions for various top tour professionals are given in table 1

Player	**V (MPH)**	**V (m/s)**	**Angle (deg)**	**Spin (rpm)**	**Spin (sec^{-1})**	**Carry (m)**
Vijay Singh	176	78.68	10.7	2,600	272.3	282.46
Robert Allenby	161	71.97	8.5	2,390	250.3	274.59
Peter Lonard	168	75.10	11.7	2,673	279.9	268.38
Phil Mickelson	178	79.57	13.0	2,200	230.4	281.64
Ernie Els	174	77.78	11.5	2,400	251.3	292.24
Test Conditions	170	76.00	12.0	2,626	275.0	????

Table 1: Lanch conditions and statistics for top PGA Tour Professionals[8]

Results

Although the amount of spin varies significantly with the skill of the player and club selection, this paper assumes as valid data taken from current tour professionals for modeling purposes. A tour pro generates about 12,000 RPM of backspin at impact when hitting with a sand wedge (loft $\approx 56°$) and anywhere from 2,200-2,800 RPM with a driver (loft $\approx 10°$). Using this information and values given in appendix A, a recursive program (see appendix B for code) was utilized to model the trajectory of a smooth golf ball in flight.

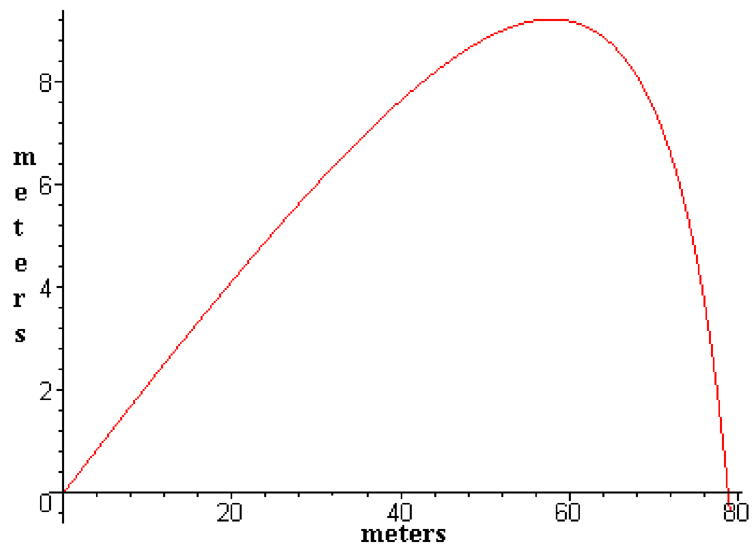

Figure 3: Simulated flight of a smooth sphere using golf tour professional launch conditions and a mathematical derivation of flight forces presented by Borg[6]

While the shape of this curve appears reasonable, anyone familiar with golf will immediately notice the short carry distance predicted in this simulation, as well as the low peak height obtained. From table (1) we could estimate that a modern golf ball launched under similar conditions would carry about 280m and reach an altitude of about 30m. These large differences between the model and observation highlight the critical role of dimples in determining golf ball trajectory.

The Effects of Dimples

Introduction

As was shown in the previous section that an idealized sphere could serve as an adequate recreational ball, in golf it is desirable to be able to make the ball fly further and allow golfers to have more control over their shot "shape" (hook or slice) and trajectory. Dimples (or their predecessors) have served this dual purpose in golf for over 100 years. While it is undisputed that dimples have had a dramatic effect on ball flight for two key reasons - lift generation and drag reduction - little progress has been made in determining which effect is more significant. In this section I attempt to show, using a simplified model of each of these effects, that drag reduction from the addition of dimples has made a significantly larger contribution to the flight of a golf ball than the addition of a lift force.

The Effect of an Additional Lift Term

Dimples generate a lift force on the golf ball because of the significant amount of backspin imparted to the ball at impact. This spin creates an asymmetry in the speeds at which air flows over the dimples in the top and bottom of the ball. Assuming the ball is traveling at velocity V_0 and the ball is spinning with angular velocity ω, the air speeds relative to the top and bottom of the ball are given by (10)

$$V_{top} = V_0 - \omega R$$

$$V_{bottom} = V_0 + \omega R \tag{10}$$

where R is the radius of ball. Since, in air, lift is proportional to velocity squared[9], this gives a net lift on the ball from the dimples of

$$Lift \propto V_{bottom}^2 - V_{top}^2 = (V_0 + \omega R)^2 - (V_0 - \omega R)^2 = 4V_0\omega R \propto V_0\omega R \tag{11}$$

Using this relation and estimates of the lift coefficient (C_L) in the range of $1.5 \text{x} 10^{-5}$ to $3.0 \text{x} 10^{-5}$ kg/m, the results in figure (4) were obtained when this term was incorporated into the existing model.

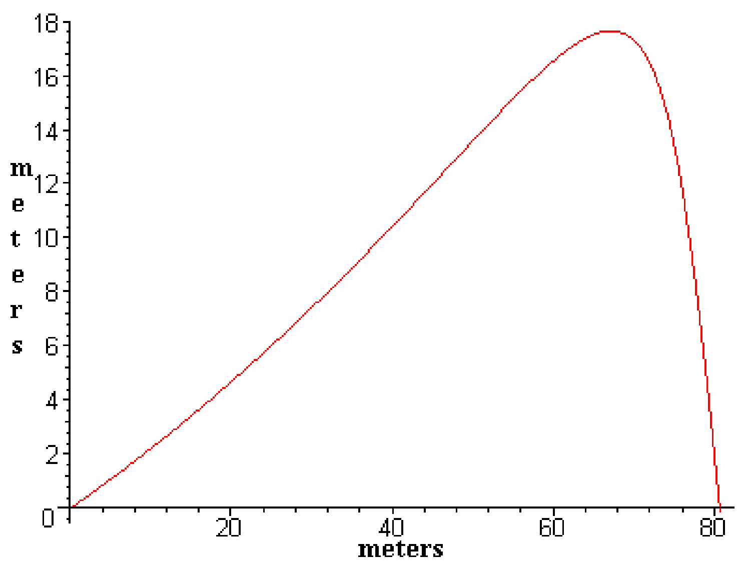

Figure 4: Simulated ball flight with the addition of a lift term

When this result is compared to the ball flight in the previous section without the lift term, it is easy to see that the lift term did little to increase the distance that the ball traveled. It should be noted, however, that the lift term did make two important changes to the result: 1) the ball traveled higher than without this term (≈9m at its peak), and 2) the ball stayed aloft for a significantly longer period of time (4.9 seconds vs. 3.1 seconds without lift). The shape of this trajectory is similar to that of a golf ball hit with more than an optimal amount of backspin - this shape is known as a "blow up" in golf (figure 5). This simulation showed several encouraging signs that seemed to indicate that the approximate lift force used gave realistic results.

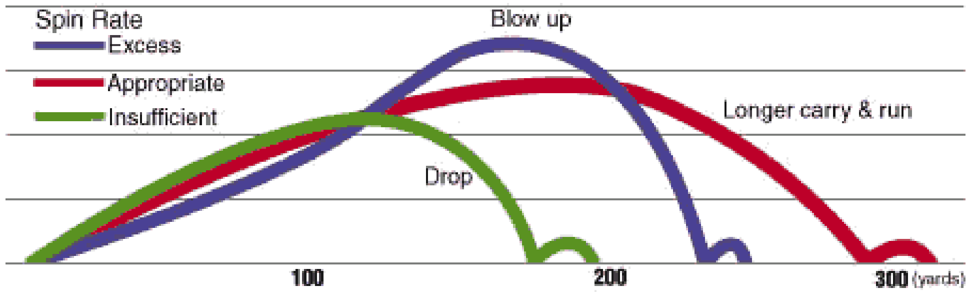

Figure 5: Observed golf ball trajectories based on spin rate at launch[10]

The Effect of Reduced Drag

After investigating the affects of the addition of lift and observing the results of its application to the model of a smooth sphere in flight it is clear that drag is still a significant factor affecting the distance that the ball carries. Dimples have a counter-intuitive effect on drag, although they roughen the surface of the ball, this adds turbulence to the boundary layer, which reduces drag by delaying boundary layer separation on the trailing side of the ball. This delayed separation of a turbulent boundary layer results in a narrower vortex street, decreasing the drag force that vortex formation exerts on the ball.

In an attempt to investigate the effects of reduced drag we make use of White's model for drag on a sphere[10]. This simplified model neglects the effect of the balls spin in its flight. The model can be summarized by equations (12) and (13)

$$C_d = \frac{8\pi}{Re \cdot \left[0.5 - \gamma + log_{10}\left(\frac{8}{Re}\right)\right]} \tag{12}$$

$$F_{drag} = -C_d R V^2 = \frac{-4\pi\nu V}{0.5 - \gamma + log_{10}\left(\frac{8}{Re}\right)} \tag{13}$$

where ν is the kinematic viscosity of air, V is the velocity of the ball, γ is the Euler-Mascheroni Constant ($\approx$.5772), R is the radius of the ball, Re is the Reynolds Number, and C_d is the drag coefficient. This model is intended to incorporate important drag considerations that the addition of dimples requires. The most important of these considerations is the alteration of the drag curve imposed by the turbulent flow created by the dimples.

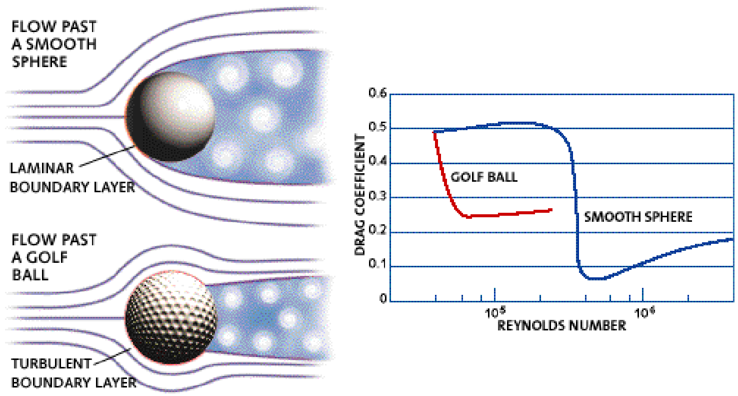

Figure 6: Flow modifications imposed by the addition of dimples[11]

This lowered Reynolds Number required for the transition to turbulent flow (steep drop in drag coefficient) is necessary because even professionals are only able to give the golf ball an initial velocity high enough to create a Reynolds Number of about $4\text{x}10^5$, which would not quite reach the value required for the large drop in drag coefficient for a smooth sphere. With the dimples, however, a Reynolds Number of only $5\text{x}10^4$ is necessary to achieve a significant drop in drag coefficient; this value can be reached by a golf ball moving at just 8.9 m/s - a speed above which the ball travels for the entirety of its flight, thereby gaining maximum possible gains from the modified drag curve.

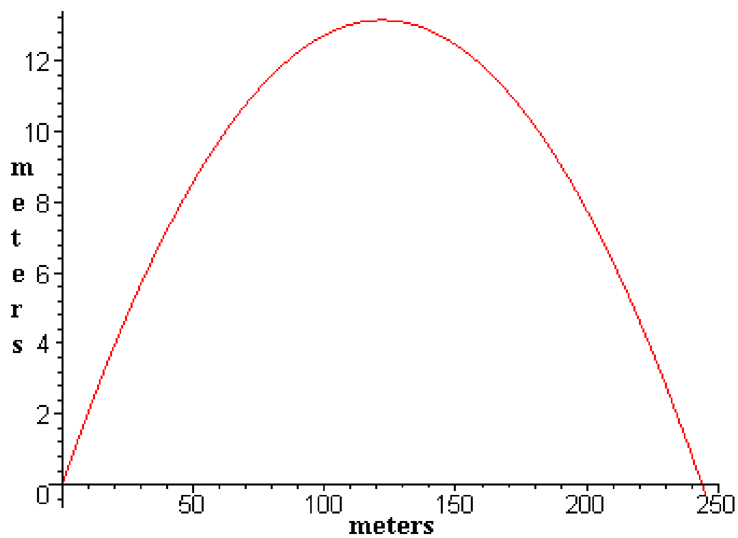

Figure 7: Simulated ball flight for a sphere with reduced drag

Several observations can be made from the results of the simulation using the simplified, reduced drag model. First, the shape of the flight is much more ideal because the model neglects the effects of spin on drag, and also excludes lift forces due to spin. The most dramatic difference to notice in this simulation is large increase in carry (over 150m). This is most likely due to a higher velocity being maintained throughout the ball's flight, which allows the ball to travel much further in the horizontal direction in the same amount of time. It is also important to note that the maximum height achieved is 60% (5 meters) higher than that of the smooth ball. While this is still quite far from the heights typical of actual golf balls (30m), it is not difficult to see the aggregate effects of the reduced drag and increased lift combining to yield a significantly higher peak height.

Combining the Effects of Additional Lift and Reduced Drag

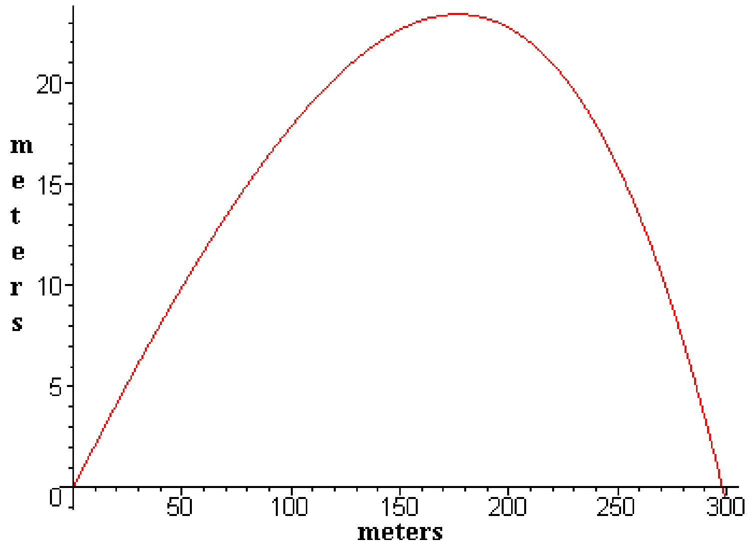

Figure 8: Simulated ball flight for a sphere with reduced drag and an additional lift term

As a final exhibition, figure (8) represents a simulated golf ball flight by using both the reduced drag force and the addition of a lift term. These two very simplistic force models combined harmoniously to produce a simulated ball flight with parameters that very nearly match observed values. An unfortunate by-product of using the simplified drag model is loss of shape in the balls path. This effect is noticeable when comparing the simplified drag model results (figures 7 & 8) to results from using Borg's full drag equation (figures 3 & 4). Both the angle of impact as well as the speed of the ball at landing should be subject to scrutiny in this model. While the ball is moving at about 37m/s just before it hits the ground in this model, observation has the ball impacting the ground at just under 32m/s. It should be noted that this is a significant improvement (in terms of modeling a golf ball) over Borg's model which predicts a smooth sphere to hit the ground at around 10m/s.

Conclusion

While it was clear that neither increasing the lift nor reducing the drag on the ball are enough alone to change the trajectory to match what is observed in actuality, it is not difficult to see that the reduction in drag has a much larger effect on the trajectory of the ball. While Borg's derivation of the forces on a smooth spinning sphere in flight is quite impressive and proves very successful in modeling the flight of a smooth sphere, a significant amount of work remains to incorporate the effects of dimples on the balls flight to a level where the model could be of use to ball designers. This appears to be a clear next step in the technological advancement of the golf industry. While practical methods and designs have brought significant advances in ball technology, computers and modern aerodynamic theory should allow for true optimization of golf ball design.

Appendix A - Constants Used for Numerical Calculations

α_τ = accommodation coefficient of tangential momentum = $\frac{360^\circ - 40^\circ}{360^\circ} = \frac{8}{9}$

R = radius of the sphere (golf ball) = 2.13cm

m = mass of a single molecule of the gas (air, average) = 28.98 amu = 4.812×10^{-26} kg

k_B = Boltzmann's Constant = $1.38 \times 10^{-23} \frac{J}{^\circ K}$

T = temperature of the gas (air) = 20°C = 293.15°K

$\vec{\omega}$ = angular velocity of sphere's rotation = 76 $\frac{rad}{sec}$ (at launch)

$\vec{\mathbf{v}}$ = velocity of the sphere (golf ball) = $76 \frac{m}{sec} \left\langle cos\left(\frac{\pi}{15}\right), sin\left(\frac{\pi}{15}\right), 0 \right\rangle$ (at launch)

Launch angle = $12^\circ = \frac{\pi}{15}$ radians

n = number density of the gas (air) = $2.503 \times 10^{25} \frac{molecules}{m^3}$

κ = heat conductivity of the sphere (assumed to be made of hard rubber) = $0.4 \frac{J}{m \cdot sec \cdot ^\circ K}$

C_p = heat capacity of the sphere = $124 \frac{cal}{^\circ K} = 519.16 \frac{J}{^\circ K}$

ρ = density of the gas (air) at 20°C and 1 atm = 1.2047 $\frac{kg}{m^3}$

μ = Absolute Viscosity of the gas (air) = $1.82 \times 10^{-5} \frac{kg}{m \cdot sec}$

Appendix B - Mathematica Code used to Model Ball Flights

```
>with(LinearAlgebra):
deltaT:=.01;
M:=.04593;
BallPosition:=<0,0,0>;
GravForce:=<0,-9.8*M,0>;

> with(LinearAlgebra):
B:=(1-I)*sqrt(VectorNorm(omega, 2)*(R^2)/(2*k));
chi:=(alphaT*n*KB*T/kappa)*sqrt(KB*T/(2*Pi*m));
k:=kappa/(rho*Cp);
z:=BesselJ(1,B)/(chi*BesselJ(1,B)+B*(BesselJ(0,B)-BesselJ(1,B)/B));
p:=n*KB*T;
xi:=1-(chi/4)*(Re(z)+sqrt(2*Pi*KB*T/m)*Im(z)/(4*VectorNorm(omega, 2)*R));

> refreshV := proc(Fnet)
global V, deltaT, M:
V:=V+ScalarMultiply(Fnet, deltaT/M):
end proc:

> refreshBallPos := proc()
global deltaT, V, BallPosition:
BallPosition:=BallPosition+ScalarMultiply(V, deltaT):
end proc:

> computeFnet := proc()
global V, omega, GravForce, B, chi, z, xi, p, k, Force, DragForce, NetForce, LiftForce,
Clift, R, CD, nu:
R:=.04267/2:
nu:=.000015:
with(LinearAlgebra);


Force:=-alphaT*(Pi/12*p*R^2*sqrt(Pi*m/(2*KB*T))*chi*Re(z)*V-2/3*xi*Pi*R^3*m*n*
CrossProduct(omega, V)-Pi/3*R^3*m*n*chi*Im(z)/VectorNorm(omega, 2)*
CrossProduct(omega, CrossProduct(omega, V))):
DragForce:=eval(Force, [alphaT=280/360, n=2.503e25, KB=1.38e-23, T=293.15,
R=.04267/2, m=4.812e-26, kappa=.4, rho=1.1347e6, Cp=519.16]):

#The following lines alter the drag force to a less precise model
#CD:=4*Pi*nu/(.5-.577215665+log10(4*nu/(R*VectorNorm(V, 2)))):
#DragForce:=ScalarMultiply(V, -CD):

#The following lines alter the drag force to the most basic model (CD=const.)
#CD:=.29;
#DragForce:=ScalarMultiply(V, -CD*R):


LiftForce:=<0,0,0>:
```

```
# The following adds a lift term to the forces
#Clift:=.000015;
#LiftForce:=Clift*CrossProduct(omega, V):

NetForce:=evalf(DragForce+GravForce+LiftForce):
end proc:

> # this subroutine updates the amount of spin on the ball (Cds is an arbitrary Cefficient)
refreshSpin := proc()
global omega, Cds, M, omegaI, deltaT:

Cds:=-24*M/(35*omegaI):
omega:=omega+ScalarMultiply(omega, deltaT*Cds*VectorNorm(omega, 2)):
end proc:

> # Main simulation routine
VI:=76;
omegaI:=275;
V:=<VI*cos(2*Pi*12/360), VI*sin(2*Pi*12/360), 0>:
StrikeAngle:=0; #defines a non-square hit that alters the spin axis, causing a hook
omega:=<0,omegaI*sin(StrikeAngle),omegaI*cos(StrikeAngle)>:
BallPosition:=<0,0,0>:
MaxInt:=1000;
with(LinearAlgebra):
Ball:=[[0,0], [0,0]]:

for i from 1 by 1 to MaxInt do

refreshBallPos():

Ball:=[op(Ball), [evalf(BallPosition[1]), evalf(BallPosition[2])]];
computeFnet():

refreshV(NetForce):
#refreshSpin():

# Stop calculating the ball path if it has gone below the ground
if ( evalf(BallPosition[2]) < 0 ) then break end if:

end do:
FlightTime:=i*deltaT;
Carry:=evalf(BallPosition[1]);
Vfinal:=evalf(V);
plot(Ball,style=LINE);
```

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